Modified phase shifting algorithm for digital holography

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In this article, we propose an extension of the Advanced Iterative Algorithm by Wang and Han. This extension enables us to calculate the amplitude distribution of the object wave directly by parameters used in the iteration process. Finally, the calculated result is compared to the measured amplitude distribution of the object wave.

1 Introduction

Phase-shifting digital holography is widely used to determine the phase as well as the amplitude of the object's wave [1, 2]. As a side effect, the zero-order term and the twin image can be eliminated in the reconstructed image.
To perform phase-shifting digital holography (PSDH), several holograms have to be recorded, each with the phase of the reference wave shifted to different states. For hologram recording a conventional holographic setup can be used, as long as a phase-shifting element is included in the optical path of the reference beam (see Fig.1).

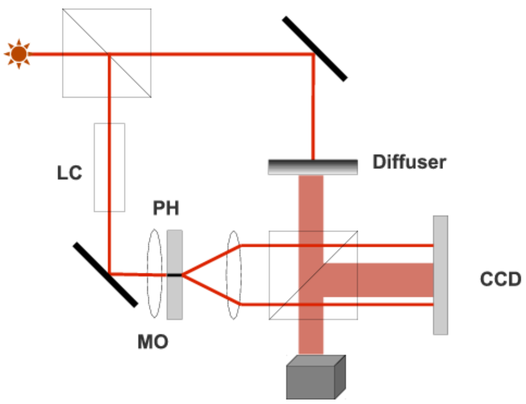

Fig. 1 *Setup for phase-shifting digital holography*

In total, there is a high number of different algorithms for PSDH available, one of them being the Advanced Iterative Algorithm (AIA) by Wang and Han [3]. A significant advantage of the AIA is, that the phase-steps do not have to be fixed. Hence, it is possible to use this method with phase-shifting elements that are not able to create repeatable steps. In this work, we will demonstrate, thatt it is possible to calculate not only the objects phases but also the amplitudes as well.

2 Advanced Iterative Algorithm

In principle, the AIA works in two steps. In the first step, the phase-values ϕ_x are considered to be known and constant, while the phase-steps δ^p have to be calculated. Hence, The intensity at the pixel number x and at the recorded hologram number p can be written as

$$I_x^p = A_x^p + C_x^p \cos\left(\phi_x + \delta^p\right) . \tag{1}$$

Using the assumption that the contrast and the intensity of the background are constant at the full area of every single hologram equation 1 can be reshaped to

$$I_x^p = a^p + b^p \cos\phi_x + c^p \sin\phi_x \tag{2}$$

$$\begin{aligned} a^p &= A_x^p \\ b^p &= C_x^p \cos\delta^p \\ c^p &= -C_x^p \sin\delta^p \end{aligned}$$

by using addition theorems. The difference between the calculated and the measured value of the intensity can be measured by applying the method of least squares to equation 2 giving

$$S^p = \sum_{x=1}^{N} \left(a^p + b^p \cos\phi_x + c^p \sin\phi_x - I_x^p\right)^2 . \tag{3}$$

Hence, the derivatives of S^p have to be set to zero

$$\frac{\partial S^p}{\partial a^p} = 0; \quad \frac{\partial S^p}{\partial b^p} = 0; \quad \frac{\partial S^p}{\partial c^p} = 0 . \tag{4}$$

The results of these three equations then can be written into a matrix

$$\begin{pmatrix} N & \sum\cos\phi_x & \sum\sin\phi_x \\ \sum\cos\phi_x & \sum\cos^2\phi_x & \sum\cos\phi_x\sin\phi_x \\ \sum\sin\phi_x & \sum\cos\phi_x\sin\phi_x & \sum\sin^2\phi_x \end{pmatrix} \times \begin{pmatrix} a^p \\ b^p \\ c^p \end{pmatrix} = \begin{pmatrix} \sum I_x^p \\ \sum I_x^p\cos\phi_x \\ \sum I_x^p\sin\phi_x \end{pmatrix} . \tag{5}$$

This system of equations has to be solved for every hologram which can be done easily because the number of holograms equals the number of phase-steps. So, δ^p can be calculated by a^p,b^p and c^p, while A^p and C^p will be dismissed.
In the second step, the phase-steps will be considered as constant and known by using the results of

the first step. Additionally, we use the assumption that the contrast and the background intensity are the same in following holograms. The new variable to be found is the phase ϕ_x. Hence, equation 1 will be reshaped to

$$\begin{aligned} I_x^p &= a_x + b_x \cos\delta^p + c_x \sin\delta^p \quad (6)\\ a_x &= A_x^p \\ b_x &= C_x^p \cos\phi_x \\ c_x &= -C_x^p \sin\phi_x \,. \end{aligned}$$

To this new equation the same procedure is applied as in the first step. So, again the final system of equations can be written in matrix-form

$$\begin{pmatrix} P & \sum\cos\delta^p & \sum\sin\delta^p \\ \sum\cos\delta^p & \sum\cos^2\delta^p & \sum\cos\delta^p\sin\delta^p \\ \sum\sin\delta^p & \sum\cos\delta^p\sin\delta^p & \sum\sin^2\delta^p \end{pmatrix} \times \begin{pmatrix} a_x \\ b_x \\ c_x \end{pmatrix} = \begin{pmatrix} \sum I_x^p \\ \sum I_x^p\cos\delta^p \\ \sum I_x^p\sin\delta^p \end{pmatrix} . \quad (7)$$

This matrix has to be solved for every pixel of the hologram which can be done easily by a LU-decomposition. With these results it is possible to gain finally gain the phase, as well as the amplitude distribution. The phase is given by

$$\phi_x = \arctan\left(-\frac{c_x}{b_x}\right) . \quad (8)$$

This result can be used as a new starting point of the first step, doing an iteration process till the result converges.

The amplitude distribution of the object wave O_x can be calculated by the three parameters a_x, b_x and c_x as well, since the intensity can be written as

$$\begin{aligned} I_x &= O_x^2 + R_x^2 + 2O_xR_x\cos(\phi_x) \quad (9)\\ &= a_x + b_x\cos\delta^p + c_x\sin\delta^p \end{aligned}$$

$$O_x = \sqrt{\frac{a_x}{2} \pm \sqrt{\frac{a_x^2 - b_x^2 - c_x^2}{4}}} . \quad (10)$$

Due to the symmetry of the solution, it is not possible to determine directly the right result of the two solutions. A possible solution of this problem is to ensure, that the reference or the object wave has a higher intensity than the other.

3 Results

In figure 2 the calculated amplitude distribution is compared with the measured one. The amplitude distribution was measured by blocking the reference wave in the optical setup.

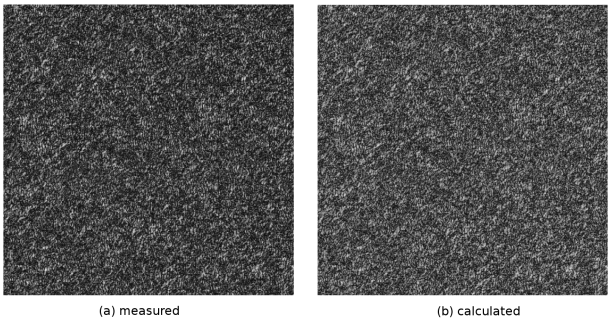

Fig. 2 *Setup for phase-shifting digital holography*

The average deviation of the two images is 1%. The reconstructed image with the calculated amplitude 3 has a slightly higher contrast than the reconstructed image with the measured amplitude. Otherwise, they are almost identical.

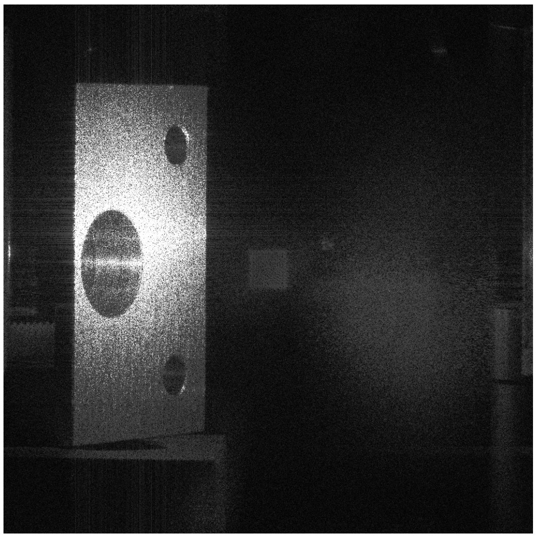

Fig. 3 *Setup for phase-shifting digital holography*

4 Conclusion

We presented a method to extend the Advanced Iterative Algorithm by Wang and Han, so that the amplitude distribution can be calculated as well. The result of the calculated amplitude distribution is almost identical to the measured one with a slightly higher contrast in the resulting reconstructed image.

References

[1] U. Schnars and W. Jueptner, *Digital Holography* (Springer Verlag 2005).

[2] I. Yamaguchi and T. Zhang, “Phase-shifting digital holography,” *Optics Letters* **22**(16), 1268-1270 (1997).

[3] Z. Wang and B. Han, “Advanced iterative algorithm for randomly phase-shifted interferograms with intra- and inter-frame intensity variations,” *Optics and Lasers in Engineering* **45**(2), 274 - 280 (2007).