# Parabasal field decomposition and its application to non-paraxial field propagation

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The propagation of harmonic fields through homogeneous media is an essential simulation technique in optical modeling and design. For non-paraxial fields common propagation techniques suffer from high numerical effort. We present a parabasal field decomposition technique which reduces the computational effort of free-space propagation by analytical handling of linear phase factors.

## 1 Introduction

Nowadays field tracing enables optical modeling and design taking the wave nature of light into consideration [1]. This approach provides the tracing of electromagnetic fields through optical systems. An essential part of this simulation technique is the propagation of harmonic fields through homogeneous media. For paraxial fields the combination of Fresnel integral and the Spectrum of Plane Waves (SPW) integral solves the problem. For non-paraxial fields the Fresnel integral cannot be applied and SPW often suffers from a too high numerical effort [2]. In some situations the far field integral can be used instead, but a general solution of the problem is not known. In the following we will present a rigorous fast fourier transformation (FFT) based propagation operator using a combination of a parabasal decomposition technique (PDT) and a semi-analytical SPW propagation operator.

## 2 Fundamentals of parabasal fields

In case of non-lossy dielectrics, a parabasal field possesses a low divergence and propagates along an arbitrary base defined by the central spatial frequency vector

$$\boldsymbol{\kappa_0} = (k_{0x}, k_{0y})^{\mathrm{T}} = k(s_{0x}, s_{0y})^{\mathrm{T}} \tag{1}$$

with the wavenumber in the homogeneous medium $k = k_0 n$. In general a parabasal harmonic field $V_\ell(\boldsymbol{\rho})$ given in a plane coordinate system $\boldsymbol{\rho}$ can be written as

$$V_\ell(\boldsymbol{\rho}) = V'_\ell(\boldsymbol{\rho})\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa_0}\cdot\boldsymbol{\rho}} \tag{2}$$

which is meaning that it is always possible to split a parabasal harmonic field into a residual field $V'_\ell(\boldsymbol{\rho})$ and a linear phase term. For a given parabasal field $V_\ell(\boldsymbol{\rho})$ we get the corresponding residual field using the shift theorem of the Fourier transformation

$$V_\ell(\boldsymbol{\rho}) = \mathscr{F}^{-1}[A_\ell(\boldsymbol{\kappa})]. \tag{3}$$

$$A_\ell(\boldsymbol{\kappa}) = A'_\ell(\boldsymbol{\kappa} - \boldsymbol{\kappa_0}) \tag{4}$$

$$V'_\ell(\boldsymbol{\rho}) = \mathscr{F}^{-1}[A'_\ell(\boldsymbol{\kappa})]. \tag{5}$$

## 3 Semi-analytical SPW propagation operator

In the previous section it was shown that a parabasal field is located around a certain spatial frequency $\boldsymbol{\kappa_0}$. Using this property we can expand $k_z$ around $\boldsymbol{\kappa_0}$ by a Taylor series

$$k_z = \gamma_0 + \boldsymbol{\gamma_1}\cdot\boldsymbol{\kappa} + \gamma(\boldsymbol{\kappa} - \boldsymbol{\kappa_0}) \tag{6}$$

where all higher order terms are included in $\gamma(\boldsymbol{\kappa} - \boldsymbol{\kappa_0})$. Plugging Eq. (6) into the rigorous SPW propagation operator [3]

$$V_\ell(\boldsymbol{\rho'}) = \mathscr{F}^{-1}[A_\ell(\boldsymbol{\kappa})\mathrm{e}^{\mathrm{i}k_z z}] \tag{7}$$

and applying the shift theorem of the Fourier transformation leads to

$$V_\ell(\boldsymbol{\rho'}, z) = V'_\ell(\boldsymbol{\rho'} + z\boldsymbol{\gamma_1}, z)\mathrm{e}^{(\mathrm{i}\gamma_0 z)}\mathrm{e}^{(\mathrm{i}(\boldsymbol{\gamma_1}\cdot\boldsymbol{\kappa_0})z)}\mathrm{e}^{(\mathrm{i}\boldsymbol{\rho'}\cdot\boldsymbol{\kappa_0})} \tag{8}$$

with the propagated shifted residual field

$$V'_\ell(\boldsymbol{\rho'}, z) = \mathscr{F}^{-1}[A'_\ell(\boldsymbol{\kappa})\mathrm{e}^{\mathrm{i}\gamma(\boldsymbol{\kappa})z}] \tag{9}$$

where the initial parabasal field is written in the seperated form $A'_\ell(\boldsymbol{\kappa})$ of Eq. (4).

## 4 Parabasal decomposition technique (PDT)

In principle the semi-analytical SPW operator can be applied to general non-parabasal harmonic fields because of the rigorous treatment of higher order phase terms in the Taylor expansion of Eq. (6). However for non-parabasal fields the sampling effort for the higher order phase terms in Eq. (9) will increase exorbitantly, resulting in a rapid increase of the numerical effort. To overcome this problem the combination of a parabasal field decomposition technique (PDT) with the semi-analytical SPW operator is used, like shown in Fig. 1.This enables the rigorous propagation of non-paraxial fields with reduced computational effort.

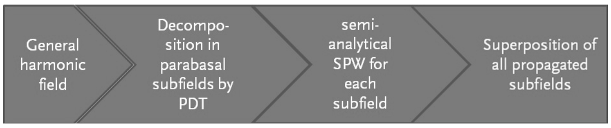


***Fig. 1*** *Flowchart for the efficient propagation of non-paraxial fields using a combination of a parabsal decomposition technique (PDT) and the semi-analytical SPW operator.*

From the physical point of view all fields can be decomposed into parabasal fields in the spatial frequency domain (like shown in Fig. 2) due to the definition of a parabasal field in the spatial frequency domain.

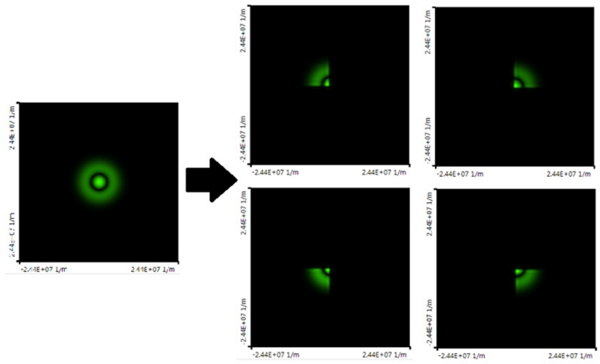

***Fig. 2*** *Decomposition of a first order Laguerre Gaussian beam into* $2\times2$ *parabasal subfields in the spatial frequency domain.*

However this is not always convenient from a numerical point of view. It is useful to distinguish between two basic cases of non-paraxial fields:

1. The field is very divergent because of small features in the field function but it can be sampled without problems in the space domain. In this case the FFT algorithm can transform the field into the spatial frequency domain, where the PDT can be applied. A Gaussian beam with small waist or a strongly scattered field are examples of such fields.
2. The field possesses a smooth but strong phase function, which does not allow its sampling in space domain. Here a FFT algorithm can not be applied, meaning that the spectrum in the spatial frequency domain is not accessible. In this case the PDT must be performed in the space domain. Such fields, e.g. a spherical or cylindrical wave usually are given in an analytical form.

## 5 Example

As an example a non-paraxial convergent spherical wave with a spherical wave radius of $R = -4$ mm is propagated by $z = 3.8$ mm, meaning that the target plane is slightly defocused. The diameter of the $x$-direction linearly polarized field is $1.28$ mm at an wavelength of $532$ nm. The initial field is decomposed in $20 \times 20$ subfields in space domain to get parabasal subfields with a sufficiently small divergence. Fig. 3 shows the residual phase of a single parabasal subfield after extracting the local linear phase.

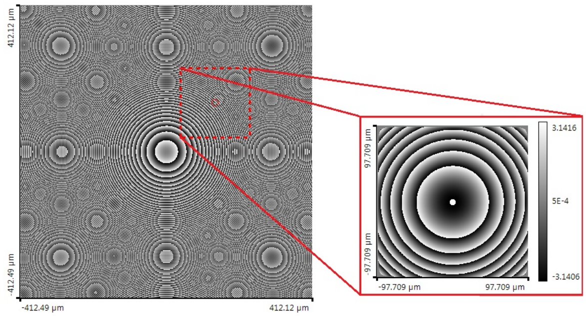

***Fig. 3*** *Parabasal field decomposition of a spherical wave in space domain.*

After applying the the semi-analytical SPW operator on each parabasal subfield and superimposing all propagated subfields the required sampling effort is given in Tab. 1.

| Method | numerical effort (sampling points) |
|---|---|
| SPW | $8585 \times 8585$ |
| PDT+ semi SPW | $98 \times 98 \times (20 \times 20)$ |

***Tab. 1*** *Sampling effort of defocused spherical wave propagation*

## 6 Conclusion

In this paper we have derived a rigorous semi-analytical SPW propagation operator. Basically the idea is an extraction of linear phase terms from the conventional SPW propagation integral kernel and its analytical handling due to the shift theorem of the Fourier transformation. The analytical handling of linear phase terms results in a significant reduction of the required sampling points. It was shown by use of an example that this semi-analytical concept allows a rigorous propagation of parabasal fields with drastically reduced numerical effort. A more detailed description can be found in [2].

## References

[1] LightTrans GmbH. VirtualLab$^{TM}$ - your optical modeling laboratory, *www.lighttrans.com* (2012)

[2] D. Asoubar, S. Zhang, F. Wyrowski and M. Kuhn: "Propagation of non-paraxial fields by parabasal field decomposition" *Proc. SPIE 8429* (2012).

[3] L. Mandel and E. Wolf: *Optical coherence and quantum optics* (Cambridge University Press, Cambridge 1995).