Comparison of phase unwrapping algorithms applied to singular phase fields

K. Seifert[1], **A. Berger**[1], **K. Mantel**[2], **V. Nercissian**[1], **N. Lindlein**[1]

[1]University of Erlangen, Institute of Optics, Erlangen, 91058, Germany
[2]Max Planck Institute for the Science of Light, Erlangen, 91058, Germany

Kristina.Seifert@physik.uni-erlangen.de

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

NATURWISSENSCHAFTLICHE FAKULTÄT

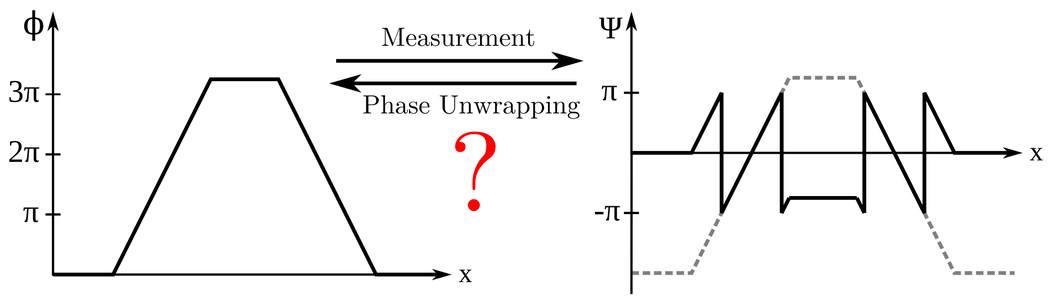
Abstract

Deformation measurements of objects with rough surfaces can be carried out using speckle interferometry. To reconstruct the surface deformation, unwrapping of the measured phase data is necessary. Due to the surface roughness, the phase data contains a high amount of singularities. To improve the evaluation of the phase data, physical means can be taken to reduce the number of singularities in the Speckle field. Nonetheless, it is inevitable to apply elaborated software algorithms for the phase unwrapping. In this work, we compare path following algorithms like Goldstein's and Flynn's algorithms, as well as a global approach like the minimum LP-Norm algorithm. Furthermore, we judge the effectiveness of above-mentioned physical means for reducing the number of residues.

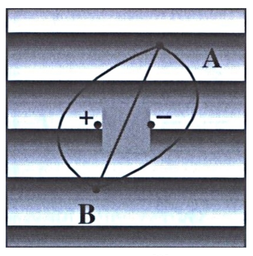
Introduction

Phase Unwrapping: attempt to reverse wrapping transformation

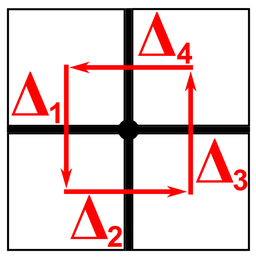
$$\Phi \rightarrow W\{\Phi\} = \Phi \mod 2\pi = \Psi$$



based on: http://www.nv.et-inf.uni-siegen.de /pb2/research/sar/phaseunw/pu_bsp.gif



source: [3]



Problems resulting from residues

- residues → unwrapped values differ depending on the chosen path
- phase field is not conservative

⇒ **need of algorithms that can handle phase fields with residues**

Identifying residues

- condition: closed loop $q = \sum_{i=1}^{4} \Delta_i \neq 0$
  ($q = 2\pi \rightarrow$ positive res., $q = -2\pi \rightarrow$ negative res.)
- Δ_i: $W\{\Psi(a) - \Psi(b)\}$ with a,b two adjacent pixels

Goldstein [1]

- searches for all residues
- connects residues to clusters by setting "branch cuts"
- total "charge" of each cluster must be 0 (same amount of pos. and neg. residues or connection to the image border)
- unwraps each resulting region

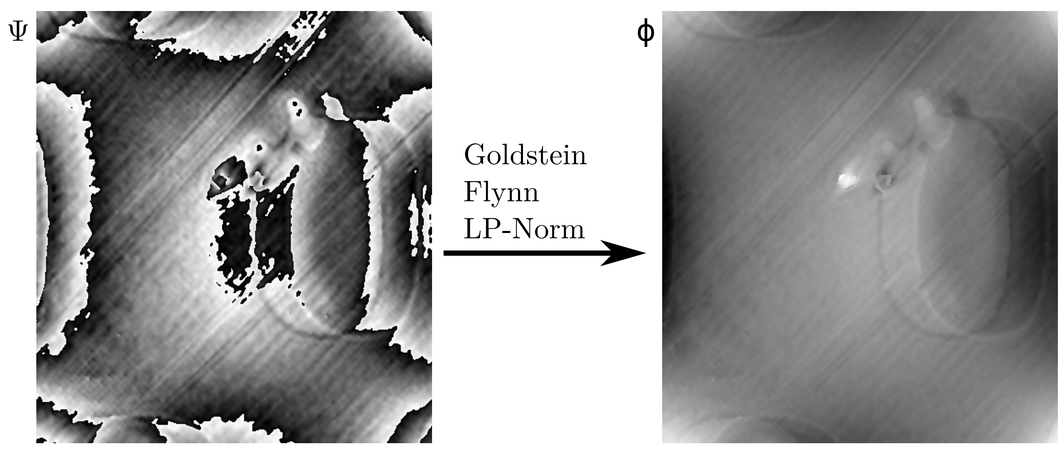
Flynn [2]

- searches for regions enclosed by fringe lines
- assigns integer multiple of 2π to each region to minimize discontinuities across the fringe lines
- minimizes sum of vertical $v = \mathrm{Int}\left(\frac{\Phi(x,y)-\Phi(x+1,y)}{2\pi}\right)$ and horizontal $h = \mathrm{Int}\left(\frac{\Phi(x,y)-\Phi(x,y+1)}{2\pi}\right)$ jump counts

LP-Norm [3]

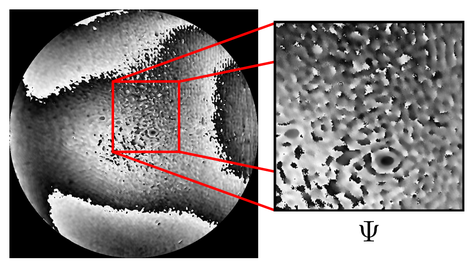
- $\Phi(x+1,y)-\Phi(x,y)$ should agree with $W\{\Psi(x+1,y)-\Psi(x,y)\}$ in the minimum L^p-Norm sense (sim. for y)
- minimizing this cost function equals solving a linear equation system with $x \cdot y$ equations
- high dimensional sparse matrix → PCG solver
- suited for parallel computing e.g. on a graphics card

Small number of residues



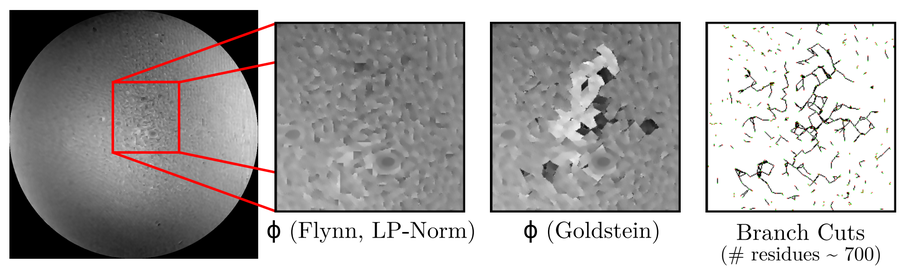
- all three algorithms succeed
- Goldstein much faster (∼ 200x)

Unfavorable distribution of residues

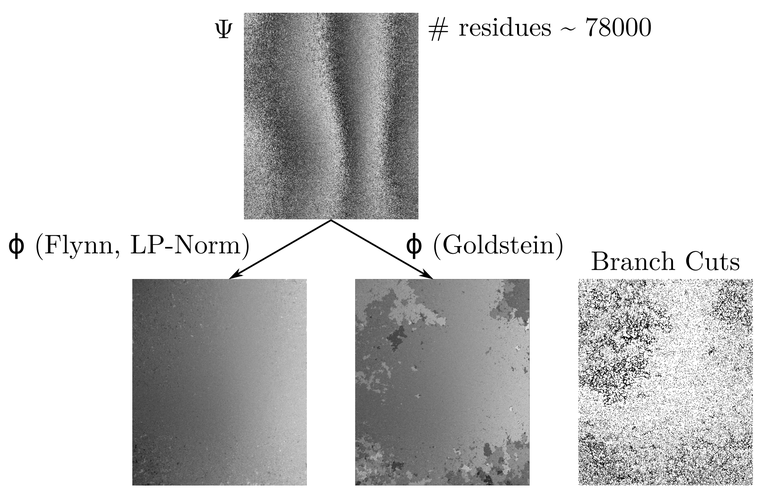


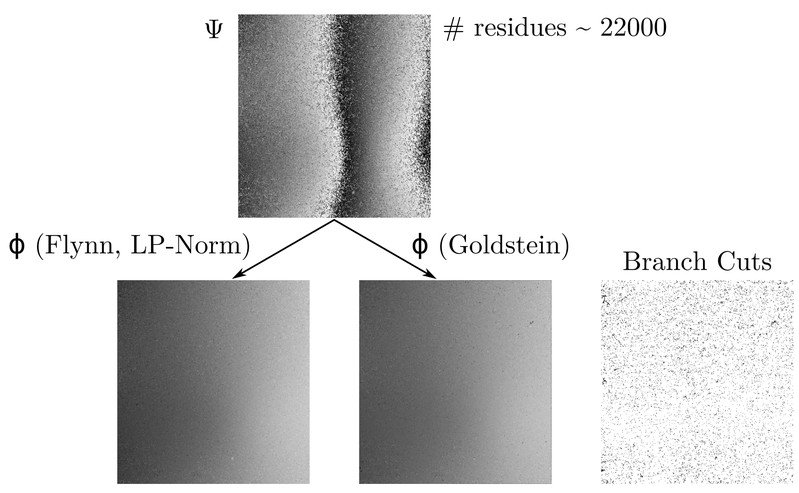
- Flynn and LP-Norm succeed
- Goldstein fails (seperates regions)

⇒ too many unfavorably distributed residues



Application to Speckle Interferometry





- number and distribution of residues important for success of Goldstein
- algorithms run faster on phase fields with less residues

⇒ **need to decrease residues count and optimize distribution of residues by physical means**

DFG-Project: MA 5167/1-1

„Reduktion von Phasensingularitäten zur Vermeidung von Unwrapping-Problemen bei der Auswertung von Speckle-Shearing-Interferogrammen"

[1] Richard M. Goldstein, Howard Zebker, Charles Werner *Satellite radar interferometry: Two-dimensional phase unwrapping*, Radio Science, Volume 23, Number 4, Pages 713-720, Juli-August 1988 .

[2] Thomas J. Flynn *Two-dimensional phase unwrapping with minimum weighted discontinuity*, JOSA A, Vol. 14, Issue 10, pp. 2692-2701 (1997) .

[3] Dennis C. Ghiglia, Mark D. Pritt *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software*, Wiley-Interscience, 1998 .