Wave front characterization of Gaussian beams using shear interferometry and a weighted reconstructor

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We present a method to recover wave fronts from a set of shear experiments. In contrast to the state of the art we take local measurement uncertainties into account when calculating the wave front from the measured finite phase differences.

1 Introduction

Shear interferometry is based upon superposition of a wave field with a shifted copy of itself. It provides great advantages for high precision wave front sensing, because it does not require a reference wave that has to be assumed known. The precision is mainly limited by the accuracy of the shear alignment and aberrations caused by the optical components of the interferometer, which can be calibrated in most cases.

The drawback of shear interferometry is that the measurement only provides finite differences $\Delta\phi(\vec{x}) = \phi(\vec{x}+\vec{s}) - \phi(\vec{x})$ of the wave front $\phi(\vec{x})$ at positions separated by the shear $\vec{s}$. Thus, the wave front has to be reconstructed from the recorded data. Usually, this task is accomplished by finding an estimate $\tilde{f} = \arg\min_f\{L\}$ that minimizes the following objective function [1]:

$$L(f) = \sum_n \sum_{\vec{x}\in R} |\Delta_n f(\vec{x}) - \Delta_n \phi(\vec{x})|^2. \qquad (1)$$

In Eq.1, the index n refers to the measurement in case that several measurements with varying shears $\vec{s}_n$ are evaluated in combination and R defines the grid of the sensing device. A number of methods exist in the state of the art to find $\tilde{f}$, of which the numerically most efficient ones apply an inverse filter in the Fourier domain. However, if the goal is accuracy rather than numerical efficiency we propose to use a different objective function

$$L_\lambda(f) = \sum_n \sum_{\vec{x}\in R} \lambda_n(\vec{x}) |\Delta_n f(\vec{x}) - \Delta_n \phi(\vec{x})|^2, \qquad (2)$$

that allows for weighting of the measured finite differences by $\lambda_n(\vec{x})$. In our investigations we have selected the weighting function proportional to the fringe modulation of the interference pattern, i.e. $\lambda_n(\vec{x}) = \gamma(\vec{x}, \vec{x}+\vec{s}_n)A(\vec{x})A(\vec{x}+\vec{s}_n)$, where A is the (real valued) amplitude of the underlying wave field and γ is the degree of coherence. This selection is justified by the fact that the fringe modulation can be regarded as a measure for the signal-to-noise ratio (SNR) of the phase measurement. A particular advantage of this choice is that the fringe modulation is inherently provided by any phase shifting approach, which means that no additional measurements are required. To find the estimate $\tilde{f}_\lambda$ that minimizes Eq.2, we employ an iterative approach based on the steepest descent gradient method [2]. Starting with an initial guess, e.g. $f^{(0)} = 0$, we iteratively improve the estimates by $f^{(k+1)} = f^{(k)} - \alpha^{(k)} \cdot \nabla L_\lambda^{(k)}$ along the negative direction given by the gradient

$$\nabla L_\lambda^{(k)}(\vec{x}) = \frac{\partial L_\lambda(f^{(k)})}{\partial f^{(k)}(\vec{x})} = 2\sum_n \psi_n(\vec{x}-\vec{s}_n) - \psi_n(\vec{x}), \qquad (3)$$

where $\psi_n(\vec{x}) = \lambda_n(\vec{x})[\Delta_n f(\vec{x}) - \Delta_n\phi(\vec{x})]$. The scalar parameter $\alpha^{(k)}$ is used to yield the minimum of L_λ along the search direction. It has to be found using a non linear search algorithm [2]. In our simulations $\alpha^{(k)}$ typically ranged from 0.3 to 1.5. The procedure is stopped when no significant change is observed in consecutive iterations.

Apart from noise dependent weighting, the method provides some further advantages over Fourier based methods, in particular:

- It can be easily adapted to arbitrary shaped pupil functions by simply setting the weighting to zero outside of the pupil area.
- The dimensions of the investigated area are not required to equal multiples of the shear.
- Shears can have any arbitrary orientation and magnitude. They do not even have to be integers of the sampling distance because the required differences in Eq. 3 can be calculated in the Fourier domain.

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2 Numerical example and conclusions

The weighting can provide great advantages in case that the measurement uncertainty shows strong spatial variations, for example due to variations of either the amplitude or the degree of coherence of the investigated wave field. We made numerical simulations to demonstrate the effect of the weighting on the reconstructed wave front. As test wave field we selected the waist of a Gaussian beam, which means that the true phase distribution is a constant, i.e. $\phi = 0$. The amplitude is designed to drop to 4% of its maximum value towards the corners of the area of investigation. The choice of a plane wave is particularly convenient for discrete Fourier based methods because it can be assumed inherently periodic.

We simulated 4 phase shifted shear experiments with the shears $\vec{s}_1, ..., \vec{s}_4$ selected to 5 and 51 pixels in either horizontal or vertical direction. For any of the shears, a sequence of 4 interference patterns has been simulated as input to a 4 frame $90°$ phase shifting algorithm. The camera simulation comprised Poisson noise assuming a full well capacity of 500 electrons and an average dark current of 4 electrons.

In Fig. 1a and Fig. 1b we see the results of the reconstructions as obtained from the simulated differences $\Delta_1\phi, ..., \Delta_4\phi$ using either a Fourier based inverse filter [3] or the iterative approach constituted by the weighted objective function Eq.2. A good measure to quantify the quality of the reconstructions, is the distance (difference) to the true wave front, which in our case is already given by the reconstructed distributions because $\phi = 0$.

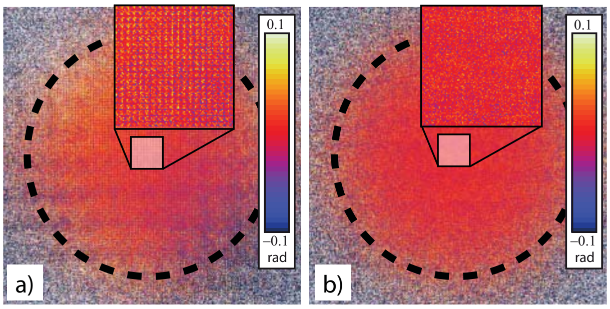

Fig. 1 *Reconstruction of the wave front across the waist of a Gaussian beam from a set of finite differences using either a) a Fourier based inverse filter or b) an iterative approach that includes noise dependent weighting. The result of b) was obtained after 200 iterations. All distributions are 510×510 pixels in size. Further details are found in the text.*

From the results we can see that both approaches provide decent reconstruction of the plane wave. This is even true for the border regions which, due to the small range of values, appear comparably noisy in Fig. 1. In case of the Fourier based inverse filter we see strong periodic deviations with periods given by the shears. This is frequency dependent noise amplification of the inversion process, owing to the roots of the corresponding shear transfer functions [3]. Due to the non-local behavior of the Fourier filter, these periodic deviations even populate the center of the wave front as seen from the detail in Fig. 1a. The standard deviation in the marked region yields $\sigma_{Fourier} = 0.026$ rad. The weighted approach prevents this error propagation from the borders to the center as seen from the detail in Fig. 1b. The standard deviation in this case is $\sigma_{Weighted} = 0.016$ rad which is an improvement by more than 2 dB.

Additionally, we see low frequent deviations in the Fourier based result that are not present in the weighted reconstruction. This is again frequency dependent noise amplification since all shear transfer functions have a root at the dc-term and comparably low SNR in close vicinity to it. However, in case of the weighted approach, errors originating from the comparably low SNR of the phase measurement in the border regions are not propagated towards the center of the reconstruction. To quantify this effect we performed a 4th-order polynomial fit to the region marked by the circle in either of the reconstructions. From the fit function we can derive a peak-to-valley value, which for the Fourier method yields $PV_{Fourier} = \pm 0.0494$ rad whereas from the weighted approach we obtain $PV_{weighted} = \pm 0.0041$ rad. This means a remarkable improvement by more than one order of magnitude due to the weighting.

We conclude that the weighting provides great advantages in the reconstruction of wave fronts from shear experiments, if the underlying wave field shows strong spatial variation in the fringe modulation. In the shown example of a Gaussian beam with the amplitude dropping to 4% towards the borders we observed a reduction of statistical deviations by more than 2 dB and a remarkable reduction of the PV-value by more than one order of magnitude.

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