Theoretical description of a method for simultaneous measurement of phase and arbitrary polarization

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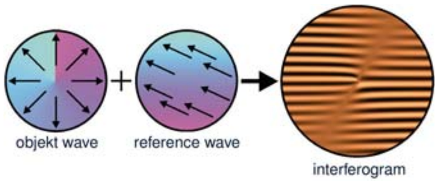
Motivation

- New interferometric measurement for simultaneous determination of arbitrary phase and polarization distributions of an object wave
- Extension of phase shifting interferometry (PSI) to polarization and phase shifting interferometry (P&PSI) with given equidistant polarization and phase steps
- Different polarization steps allow calculation of object polarization



Theory

- General description of polarized light by Jones vector:

$$\vec{J} = \begin{pmatrix} E_{0x}e^{i\delta_x(x,y)} \\ E_{0y}e^{i\delta_y(x,y)} \end{pmatrix} e^{i\Phi'(x,y)} \Rightarrow \vec{J} = E(x,y) \begin{pmatrix} cos(\alpha(x,y)) \\ sin(\alpha(x,y))e^{i\delta(x,y)} \end{pmatrix} e^{i\Phi(x,y)}$$

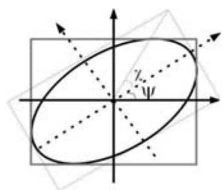
with $\Phi' = \Phi - \delta_x$, $\delta = \delta_y - \delta_x$ and $\alpha = \arctan(\frac{E_{0y}}{E_{0x}})$

- Object beam: phase Φ and polarization states described by α and δ:

$$\vec{J_O} = E_O(x,y) \begin{pmatrix} cos(\alpha(x,y)) \\ sin(\alpha(x,y))e^{i\delta(x,y)} \end{pmatrix} e^{i\Phi(x,y)}$$

- Reference beam: In the setup the different polarization states are set by a combination of a half-wave plate (HWP) and a quarter-wave plate (QWP). It's appropriate to characterize the reference Jones vector by the orientation Ψ and the ellipticity χ of the polarization ellipse:

$$\vec{J_R} = E_R(x,y)\frac{-1}{\sqrt{2}} \begin{pmatrix} cos(\Psi + 2\chi) - i\cos(\Psi) \\ \sin(\Psi + 2\chi) - i\sin(\Psi) \end{pmatrix} e^{i\varphi(x,y)}$$



Ψ: orientation, χ: ellipticity

HWP rotated by $\frac{\Psi}{2}$

QWP rotated by $\Psi + \chi$

- Intensity pattern (analog to Michelson interferometric formula):

$$I = |\vec{J_O} + \vec{J_R}|^2 = I_0[1 - \frac{V}{\sqrt{2}}(\cos(\alpha)[\cos(\Phi - \varphi)\cos(\Psi + 2\chi) - \cos(\Psi)\sin(\Phi - \varphi)] + \sin(\alpha)[\sin(\Psi + 2\chi)\cos(\Phi - \varphi + \delta) - \sin(\Psi)\sin(\Phi - \varphi + \delta)])]$$

Measurement method

- The intensity I_j is measured for each polarization and phase step (Ψ_j, χ_j, φ_j). Rewrite the intensity into a vector product

$$I_j = \vec{Y}^T\vec{X}_j \qquad \text{(with} \quad a_{p/n} = \Phi \pm \alpha; \quad e_{p/n} = \Phi \pm \alpha + \delta)$$

- Thereby $\vec{Y}$ consists of all unknown parameters $(I_0, V, \Phi, \alpha, \delta)$ whereas $\vec{X}$ includes the adjustable reference values $(\Psi_j, \chi_j, \varphi_j)$
- With the matrix $\hat{A}$ the equation can be solved for $\vec{Y}$:

$$\vec{Y} = \hat{A}^{-1}\sum_{j=1}^{N}\vec{X}_j I_j; \qquad \text{(with} \quad \hat{A} = \sum_{j=1}^{N}\hat{A}_j = \sum_{j=1}^{N}\vec{X}_j\vec{X}_j^{\,T})$$

Only for specified values of $\Psi_j \in [0;\pi]; \chi_j \in [-\pi/4;\pi/4], \varphi_j \in [0;2\pi]$ $\hat{A}$ is invertible.

$$\vec{Y} = \frac{-I_0V}{4\sqrt{2}} \begin{pmatrix} -\frac{4\sqrt{2}}{V} \\ -\sin(a_n) - \sin(a_p) + \sin(e_p) - \sin(e_n) \\ -\sin(a_n) - \sin(a_p) - \sin(e_p) + \sin(e_n) \\ \cos(a_n) + \cos(a_p) + \cos(e_n) - \cos(e_p) \\ \cos(a_n) + \cos(a_p) - \cos(e_n) + \cos(e_p) \\ \cos(a_n) + \cos(a_p) + \cos(e_n) - \cos(e_p) \\ \cos(a_n) + \cos(a_p) - \cos(e_n) + \cos(e_p) \\ \sin(a_n) + \sin(a_p) - \sin(e_p) + \sin(e_n) \\ \sin(a_n) + \sin(a_p) + \sin(e_p) - \sin(e_n) \end{pmatrix} \quad \vec{X}_j = \begin{pmatrix} 1 \\ \cos(\varphi_j - \Psi_j) \\ \cos(\varphi_j + \Psi_j) \\ \sin(\varphi_j - \Psi_j) \\ \sin(\varphi_j + \Psi_j) \\ \cos(\varphi_j - (\Psi_j + 2\chi_j)) \\ \cos(\varphi_j + (\Psi_j + 2\chi_j)) \\ \sin(\varphi_j - (\Psi_j + 2\chi_j)) \\ \sin(\varphi_j + (\Psi_j + 2\chi_j)) \end{pmatrix}$$

$$I_0 = Y_1; \quad \Phi + \delta = \arctan(\tfrac{Y_3 - Y_2}{Y_4 - Y_5}); \quad \Phi = \arctan(\tfrac{Y_8 + Y_9}{Y_6 + Y_7});$$

$$\alpha = \arctan(\sqrt{\tfrac{(Y_3-Y_2)^2+(Y_4-Y_5)^2}{(Y_8+Y_9)^2+(Y_6+Y_7)^2}}); \quad V = \frac{\sqrt{(Y_3-Y_2)^2+...+(Y_6+Y_7)^2}}{Y_1}$$

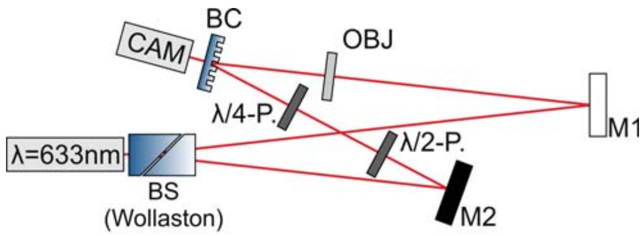
$\alpha \in [0, \frac{\pi}{2}]$, $\delta \in [0, \pi]$ and $\Phi \in [0, 2\pi]$

- Different evaluation algorithms are possible. For each the polarization and phase steps are equidistant and there must be at least $N \geq 9$ images.
  - "Non averaging" algorithm: phase steps: $\delta_\varphi = \frac{2\pi}{N_\varphi}$, possible for $N_{\Psi,\chi} \geq 2$ and $N_\varphi \geq 3$
  - "Averaging" algorithm: phase steps: $\delta^*_\varphi = \frac{2\pi}{N_\varphi - 1}$, possible for $N_\chi \geq 2$, $N_\Psi \geq 3$ and $N_\varphi \geq 4$

$\mapsto$ **results in Poster Nr.2**

Measurement setup

- Folded Mach-Zehnder interferometer allows almost normal (5°) incidence on the mirrors, so influences on the polarization are suppressed.
- With a Wollaston prism as beamsplitter (BS) the intensity ratio between object and reference arm is adjustable and therefore the visibility of the interferogram can be improved.
- Polarization shifting is enabled by rotating the HWP and QWP in the reference arm. For phase shifting mirror M1 in the object arm is moveable.
- To conserve the incident polarization distributions a special phase grating is used as beam combiner (BC).



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