Unification of the Geometric and Diffractive Theories of Electromagnetic Fields

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In physical optics we deal with electromagnetic fields, which are governed by Maxwell's equations. In order to solve them fast we couple different Maxwell solvers in a non-sequential field tracing concept. Further pillars of a fast physical optics concept are: (1) Solving Maxwell's equations in the k-domain whenever possible. (2) Switching between k and space domains by either a regular or geometric Fourier transform, depending in which field zone we are. (3) Modeling the effect of optical components by so-called bidirectional operators. (4) Introduction of geometric bidirectional operators. The combination of these concepts results in a physical optics theory, with a fast modeling algorithm, which inherently applies geometric and diffractive models in a well-defined, cogent way.

1 Field tracing diagram

The solution of Maxwell's equations for an optical system can be obtained through a non-sequential field tracing algorithm [1]. This leads to the modeling of various optical paths through the system, all of them consisting of a sequence of free-space propagation steps and interactions with inhomogeneous regions in space, e.g. optical components. Starting from a field in the source plane, the free-space operators $\boldsymbol{\mathcal{P}}$ provide the fields at the next component plane where the component response is evaluated by an operator $\boldsymbol{\mathcal{B}}$. These operations can be performed in the x or the k domain. The modeling of one lightpath can be illustrated in a so-called field tracing diagram. An example is depicted in Fig. 1.

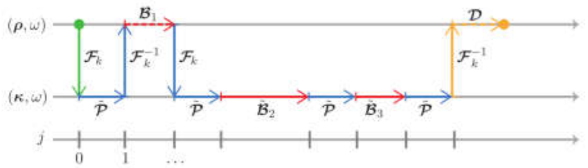

Fig. 1 *Illustration of the physical optics modeling of a lightpath by a field tracing diagram. The index j indicates the field reference planes, where the operators are applied.*

Though the electromagnetic field consists of six field components, the field tracing algorithm can be formally expressed (with $\boldsymbol{\rho} = (x, y)$) by $\boldsymbol{E}_\perp(\boldsymbol{\rho},\omega) = (E_x(\boldsymbol{\rho},\omega), E_y(\boldsymbol{\rho},\omega))$ only, since the missing four components can be calculated on demand from $\boldsymbol{E}_\perp$. In the k domain these calculations follow from simple algebraic equations.

The free-space operator equation is given by $\boldsymbol{E}_\perp^{\text{out}}(\boldsymbol{\rho},\omega) = \boldsymbol{\mathcal{P}}\boldsymbol{E}_\perp^{\text{in}}(\boldsymbol{\rho},\omega)$ with the field in the input plane $\boldsymbol{E}_\perp^{\text{in}}$ and the resulting field in the output plane (which is the input plane of the next operator) $\boldsymbol{E}_\perp^{\text{out}}$. The propagation operator $\boldsymbol{\mathcal{P}}$ formally expresses the free-space propagation by diffraction integrals and an additional tilt operator if the in/out-planes are not parallel [2]. Though in the space domain the propagation is expressed by diffraction integrals with high numerical cost, in the k domain we have the simple product ($\boldsymbol{\kappa} = (k_x, k_y)$)

$$\begin{aligned}\tilde{\boldsymbol{E}}_\perp^{\text{out}}(\boldsymbol{\kappa},\omega) &= \tilde{\boldsymbol{\mathcal{P}}}\tilde{\boldsymbol{E}}_\perp^{\text{in}}(\boldsymbol{\kappa},\omega)\\ &= \exp\big(\mathrm{i}\check{k}_z(\boldsymbol{\kappa})\Delta z\big) \times \tilde{\boldsymbol{E}}_\perp^{\text{in}}(\boldsymbol{\kappa},\omega) \end{aligned} \quad (1)$$

for parallel planes and additional coordinate transformations for non-parallel planes. By the selection of a regular or a geometric Fourier transform [3] to switch between the k and space domains, different diffraction integrals follow in space domain from Eq. 1, including the Rayleigh-Sommerfeld, the far-field, and the Debye integral. The simplicity of free-space propagation in the k domain is one important reason why the k domain is the preferred domain of fast physical optics. The option of a fast algebraic calculation of $\tilde{E}_z$ and $\tilde{\boldsymbol{H}}$ from $\tilde{\boldsymbol{E}}_\perp$ is another reason. Let us next turn to the B operators in the field tracing diagram.

2 Bidirectional operators

In space domain we have the B operator $\boldsymbol{E}_\perp^{\text{out}}(\boldsymbol{\rho},\omega) = \boldsymbol{\mathcal{B}}\boldsymbol{E}_\perp^{\text{in}}(\boldsymbol{\rho},\omega)$ and analogously in the k domain $\tilde{\boldsymbol{E}}_\perp^{\text{out}}(\boldsymbol{\kappa},\omega) = \tilde{\boldsymbol{\mathcal{B}}}\tilde{\boldsymbol{E}}_\perp^{\text{in}}(\boldsymbol{\kappa},\omega)$. In both domains the operators have a matrix from, e.g. in k domain

$$\begin{pmatrix}\tilde{E}_x^{\text{out}}(\boldsymbol{\kappa},\omega)\\ \tilde{E}_y^{\text{out}}(\boldsymbol{\kappa},\omega)\end{pmatrix} = \begin{pmatrix}\tilde{\mathcal{B}}_{xx} & \tilde{\mathcal{B}}_{xy}\\ \tilde{\mathcal{B}}_{yx} & \tilde{\mathcal{B}}_{yy}\end{pmatrix}\begin{pmatrix}\tilde{E}_x^{\text{in}}(\boldsymbol{\kappa},\omega)\\ \tilde{E}_y^{\text{in}}(\boldsymbol{\kappa},\omega)\end{pmatrix}. \quad (2)$$

Each operator in this matrix represents an integral operator, e.g. in the k domain we have integrals of the form (skipping ω)

$$\tilde{V}^{\text{out}}(k_x, k_y) = \int\int_{\mathsf{K}^2} \tilde{B}(k_x, k_y, k_x', k_y')\tilde{V}^{\text{in}}(k_x', k_y')\,\mathrm{d}k_x'\,\mathrm{d}k_y', \quad (3)$$

DGaO-Proceedings 2017 – http://www.dgao-proceedings.de – ISSN: 1614-8436 – urn:nbn:de:0287-2017-A036-9

submitted: 25.Aug.2017 published: 11.Sep.2017

where K^2 denotes the set of k values of the input component. $\tilde{V}$ is a placeholder for the field components and $\tilde{B}$ expresses the integral kernel of one of the matrix components in Eq. 2. Since (k_x, k_y) represents the direction of the propagating plane waves in the k domain, in a subset of K^2 the kernel $\tilde{B}(k_x, k_y, k'_x, k'_y)$ can also be understood as a function of direction angles, which indicates that $\tilde{B}$ is a generalization of the Bidirectional Scattering Distribution Function (BSDF) for electromagnetic fields, whereas the BSDF formulates the effect on the field energy only.

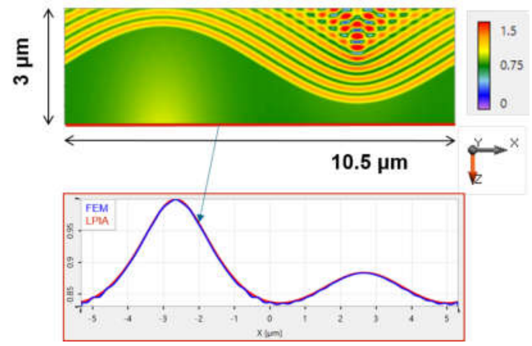

Fig. 2 *In the upper part the field inside a sinusoidal surface grating is shown. It was calculated with a Finite Element Technique (FEM). Alternatively the Local Plane Interface Approximation (LPIA) was used. The resulting field amplitude in the plane which is marked red is shown in the lower figure for both techniques. Courtesy of Rui Shi.*

This is of course included in Eq. 3. Because of this relation to the BSDF we refer to $\tilde{\boldsymbol{\mathcal{B}}}$ as a bidirectional operator or simply the B operator. In general the evaluation of $\tilde{B}(\boldsymbol{\kappa}, \boldsymbol{\kappa}')$ and its application to the calculation of the integral in Eq. 3 is a numerically heavy task and not fast. However, in case of stratified media we have the simple form $\tilde{B}(\boldsymbol{\kappa}, \boldsymbol{\kappa}') = \tilde{B}(\boldsymbol{\kappa})\delta(\boldsymbol{\kappa} - \boldsymbol{\kappa}')$, which reduces the integral to a multiplication and enables a fast evaluation of the operator in the k domain [4]. If we consider the effect of an aperture in terms of Kirchhoff's boundary condition, then the operator B in space domain is in the form of a simple factor, which leads us to model this effect in the x domain by a suitable selection of Fourier transforms, which is indicated by the first B operator in Fig. 1. Of course, the major task in optics is the propagation of a field through a general surface between two media, e.g. for lens modeling.

3 Geometric operators

The effect of a general surface on a field can be evaluated by for example a Finite Element Method (FEM), however for most situations the numerical effort is too high. If the structures on the surface are not too small, the B operator can be modeled with an accuracy which is sufficient for most practical situations by the so-called Local Pane Interface Approximation (LPIA) [5]. In this approximation the boundary conditions of the electromagnetic fields are solved locally by application of the known solution for stratified media. In Fig. 2 a comparison between FMM and LPIA is shown for a sinusoidal surface grating, which shows that LPIA predicts the effect well, even for quite small features in the surface. In fact, we found that LPIA is a powerful technique to evaluate $\tilde{B}(\boldsymbol{\kappa}, \boldsymbol{\kappa}')$ of Eq. 3 including vectorial effects (see Eq. 2). It should be noted that the well-known thin element approximation (TEA) is a simplified special case of LPIA. Though LPIA enables the evaluation of the bidirectional operator, we still have to implement the numerically heavy integral of Eq. 3. That leads to the combination of LPIA with the geometric Fourier transform [3]. If we assume that the incident $\boldsymbol{E}_\perp^{\text{in}}$ and the output field $\boldsymbol{E}_\perp^{\text{out}}$ are in its geometric field zones, then it follows via the theory of the geometric Fourier transform that

$$\tilde{B}^{\mathsf{LPIA}}(\boldsymbol{\kappa}, \boldsymbol{\kappa}') = \tilde{B}^{\mathsf{LPIA}}(\boldsymbol{\kappa})\delta\big(\boldsymbol{\kappa} - \boldsymbol{\kappa}(\boldsymbol{\kappa}')\big) \tag{4}$$

and the integral in Eq. 3 reduces to a simple multiplication again which includes a coordinate transformation $\boldsymbol{\kappa}(\boldsymbol{\kappa}')$. This transformation follows from the wavefront phase of the input field according to the theory of the geometric Fourier transform. We refer to the operator in Eq. 4 as a geometric operator. This result has been implemented in VirtualLab Fusion [6]. It can be numerically tested in any plane if the field is in its geometric or diffractive zone. Depending on the result different Fourier transforms are applied and the B operators are applied in different ways. This results in an inherent application of diffractive and geometric models based on pure mathematical arguments. The modeling is always fully based on physical optics and optimized in numerical efficiency.

References

[1] M. Kuhn, F. Wyrowski, and C. Hellmann,in vol. 66 of *Lecture Notes in Applied and Computational Mechanics*, pp. 257 (Springer Berlin Heidelberg, 2013).

[2] S. Zhang, D. Asoubar, C. Hellmann, and F. Wyrowski, Appl. Opt. **55**(3), 529 (2016).

[3] F. Wyrowski and C. Hellmann, *Proc. DGaO*, vol. 118, A37 (2017).

[4] S. Zhang, C. Hellmann, and F. Wyrowski, Appl. Opt.**56**(15), 4566 (2017).

[5] A. Pfeil, F. Wyrowski, A. Drauschke, and H. Aagedal, Appl. Opt. **39**(19), 3304 (2000).

[6] Fast physical optics software "Wyrowski VirtualLab Fusion", developed by Wyrowski Photonics UG, distributed by LightTrans GmbH, Jena, Germany.