

Compensation of higher order rotationally symmetric Zernike coefficients caused by the defocusing of spherical sections

Markus Schake

Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany

mailto:markus.schake@ptb.de

In interferometric measurements of sphericity, by comparison of the sample under test to a spherical reference wavefront, defocusing gives rise to spherical aberrations represented by higher order, rotationally symmetric Zernike coefficient Z_4^0 , Z_6^0 and so on. Since in practical applications it is often not possible to verify the absolute position of the sample under test with respect to the reference transmission sphere, these aberrations have to be characterized and corrected in the measured topography. This article gives a tutorial like instruction of how to model, identify and correct these aberrations.

1 Introduction

The term sphericity refers to the characteristic of a sample under test to resemble a perfect sphere of arbitrary diameter. The deviation of the samples surface form, at each point of the topography, in comparison to a perfect sphere defines its sphericity. The sphericity of imaging lenses is used in the assessment of their aberrations and is an important tool for quality control of optical components. Multiple methods for the absolute and relative determination of sphericity have been proposed in literature [1, 2, 3, 4, 5, 6, 7].

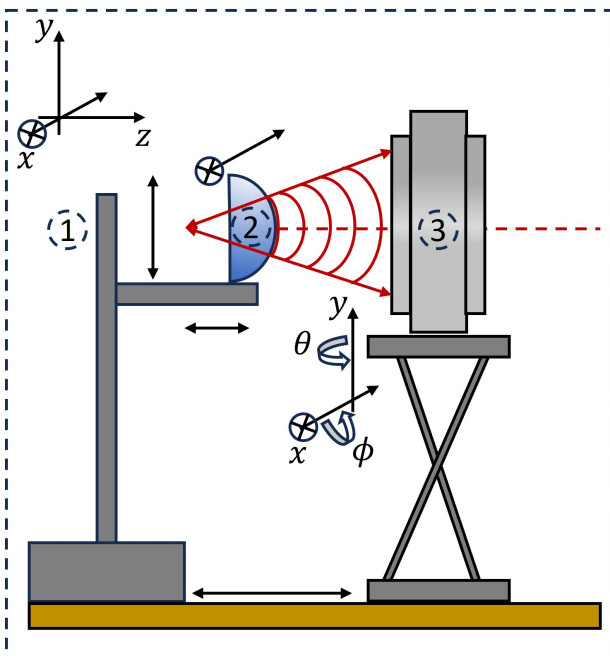


Fig. 1 Cavity between the sample under test (2) on the sample mount (1) and the transmission sphere (3) in the confocal position $z = z_{\text{conf}}$. The interferometer is placed to the right of the transmission sphere but is not depicted here.

The PTB offers calibration for the sphericity of spher-

ical sections of concave and convex lenses and mirrors with an expanded measurement uncertainty of the sphericity at each point of the topography of $U_s = 11 \text{ nm}$ with coverage factor $k_p = 2$ for normal distributed measurement errors. The measurement setup is a common path Fizeau interferometer, which compares the calibrated reference wavefront W_{TS} of a transmission sphere to the sample under test, who's form is represented by the wavefront W_{sut} , in a confocal null configuration. The confocal measurement principle is depicted in Fig.1. The sphericity calibration of spherical sections at PTB involves a focus scan of the sample under test, during which the topography of the sample under test is retrieved at multiple points close to the ideal confocal position $z = z_{\text{conf}}$. The sample under test (2) has a constant radius of curvature R_{sut} , while the radius of curvature $R_{\text{WTS}}(z)$ of the impinging reference wavefront from the transmission sphere (3) varies with its propagation along the optical axis. In the confocal position the curvature radius of the sample and the curvature radius of the reference wavefront align perfectly $R_{\text{sut}} = R_{\text{WTS}}(z)|_{z_{\text{conf}}}$. At all positions outside the confocal position the curvature radius of sample and reference wavefront are different $R_{\text{sut}} \neq R_{\text{WTS}}(z)|_{z \neq z_{\text{conf}}}$. The interferometer is measuring the wavefront difference $W_{\text{sut}} - W_{\text{TS}}$, which considering W_{TS} to be a perfectly spherical wave defines the sphericity of the sample under test. The sphericity is commonly represented by the $N_z = 36$ coefficients of a Zernike polynomial fitting. The Zernike coefficient for defocus $Z_2^0 = 2R^2 - 1$ with R being the radius of curvature of the spherical part of the fitted wavefront, describes a full hemisphere above the unit circle. If spherical wavefronts with slightly different radius of curvature, like R_{sut} and $R_{\text{WTS}}(z)$ are compared by scanning around the confocal position $z = z_{\text{conf}}$ in the same limited aperture, only one or neither of them may represent a full hemisphere over the chosen region of interest.

Thus, spherical sections, limited by the aperture diameter are compared and their difference $W_{\text{sut}} - W_{\text{TS}}$, which forms the interferogram, does contain higher order rotationally symmetric Zernike terms and may not be described by pure defocus Z_2^0 .

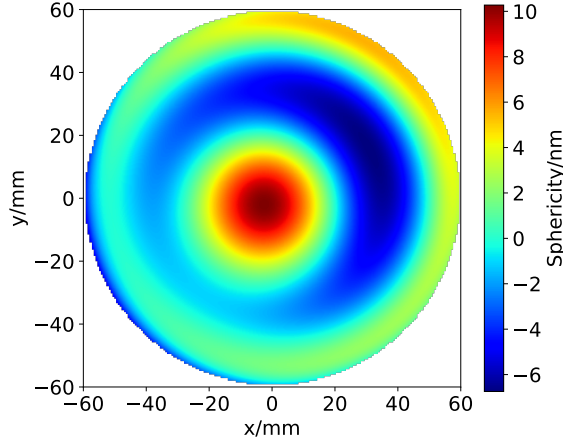


Fig. 2 Deviation from sphericity.

The Zernike terms are enumerated following the Zernike Fringe Wyant (ZFR) [8] enumeration scheme, including a subset of $N_Z = 36$ Zernike polynomials with:

- ZFR3 = Z_2^0 Defocus
- ZFR8 = Z_4^0 Spherical aberration
- ZFR15 = Z_6^0 High order spherical aberration

Consider a spherical sample under test specified by $R_{\text{sut}} = 100$ mm, a free aperture diameter of $D_{A,\text{sut}} = 120$ mm and a topography error as depicted in Fig. 2 specified by the Zernike coefficients ZFR4 = 1 nm, ZFR6 = 3 nm, ZFR7 = 4 nm, ZFR8 = 5 nm and ZFR15 = -5 nm.

Fig. 3 demonstrates the evolution of selected Zernike coefficients of the retrieved sphericity while the sample under test is moved through the focus region along the optical axis at positions $(z - z_{\text{conf}})$. It shows that the determined value of the rotationally symmetric Zernike terms Z_4^0 and Z_6^0 have a dependence of the distance to the focus position, while rotationally variant features of the topography like ZFR4 = Z_2^2 , ZFR6 = Z_3^1 and ZFR7 = Z_3^{-1} stay constant. The actual topography features are those associated with the focal position $z = z_{\text{conf}}$ for which the defocus term Z_2^0 becomes 0. Since it is not suitable in practical applications to have the sample perfectly aligned in the confocal position during measurement, multiple interferograms with different defocus are recorded and the value of Z_4^0 and Z_6^0 is determined by linear interpolation as might be derived from Fig. 3.

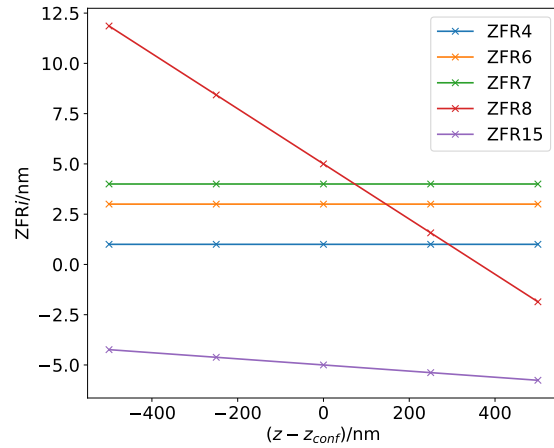


Fig. 3 Retrieved Zernike Polynomial coefficients during focus scan.

This enables the robust determination of sphericity with low measurement uncertainties. The impact of defocus on the measurement results depends on R_{sut} and $D_{A,\text{sut}}$. Steeper slopes on the surface cause higher sensitivity to defocus. The results presented in this article are generated with a python script, which is freely available at PTB's open access repository identified by the citable DOI:10.7795/710.20240403.

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