

The anycentric camera: A unifying pinhole model for ento-, hyper- and telecentric lenses using Geometric Algebra

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Camera calibration usually relies on the well-known pinhole camera model [1], which, however, only applies to the widely used entocentric lenses, whereas optical imaging and metrology systems often use telecentric or sometimes even more exotic hypercentric lenses. It is shown how the pinhole model can be generalized to all these lenses using the unifying methods of Geometric Algebra [2].

1 Introduction

The widely used *pinhole camera model* [1] has proved useful for camera calibration due to its relatively small number of parameters, yet accurate description of the system behavior. However, it is only suitable for the commonly used *entocentric* lenses, whose object-sided chief rays *converge* towards the camera (Fig. 2). This makes objects further away from the camera appear smaller in the camera image (Fig. 1). Nevertheless, other lens types are used especially in optical imaging and metrology systems to solve otherwise complex problems: *Telecentric* lenses produce the same image of an object regardless of its distance from the camera (Fig. 1). This is achieved by *parallel* chief rays (Fig. 2). *Hypercentric* lenses provide *diverging* chief rays towards the camera (Fig. 2), facilitating a simultaneous inspection of the top and side surfaces of an object (Fig. 1).

2 State of the Art

At present, no single model can account for these fundamentally different chief ray paths (Fig. 2): For entocentric lenses, the well-known pinhole model is typically employed using the center of the entrance pupil as the so-called *camera center*. The distance between the camera center and the image plane is

referred to as the *camera constant* c . For telecentric lenses, the current pinhole model cannot be applied because their entrance pupil respectively camera center is at infinity. For hypercentric lenses, the pinhole model must be modified by introducing a negative camera constant [3].

3 Anycentric pinhole camera model

Instead, the following “*anycentric*” camera model proposed in [4] is equally applicable to ento-, tele- and hypercentric lenses (Fig. 3). It is facilitated by the unifying methods of *Plane-based Geometric Algebra (PGA)* [5,6] $\mathbb{R}_{n,0,1}^*$ with $e_i^2 = 1$ for $i = 1, 2, \dots, n$ and $e_0^2 = 0$, describing the *Euclidean space* with dimension n . Constructing the model can be broken down into the following three steps. The camera center is called c , the algebra’s *duality operator* is denoted as “ $*$ ”.

1. Coordinate system:

The origin of the camera coordinate system is conventionally placed in the camera center. However, since for telecentricity it should be possible to locate the camera center at infinity, in the new model it is placed in the *image* with the n^{th} -axis as optical axis. For the ento- and hypercentric cases, it can be easily converted back to the classical model by the translation *motor* $\sqrt{c/e_0}$.

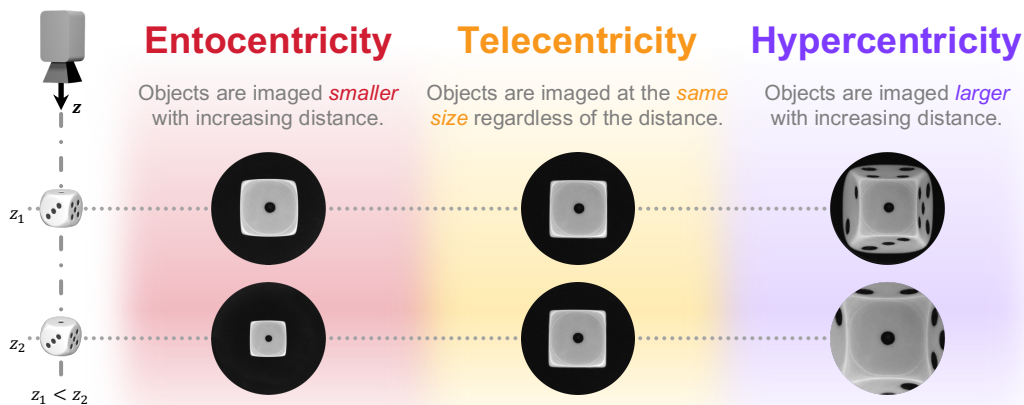


Fig. 1 Images of a dice at different distances from the camera with entocentric, telecentric and hypercentric lenses. [4]

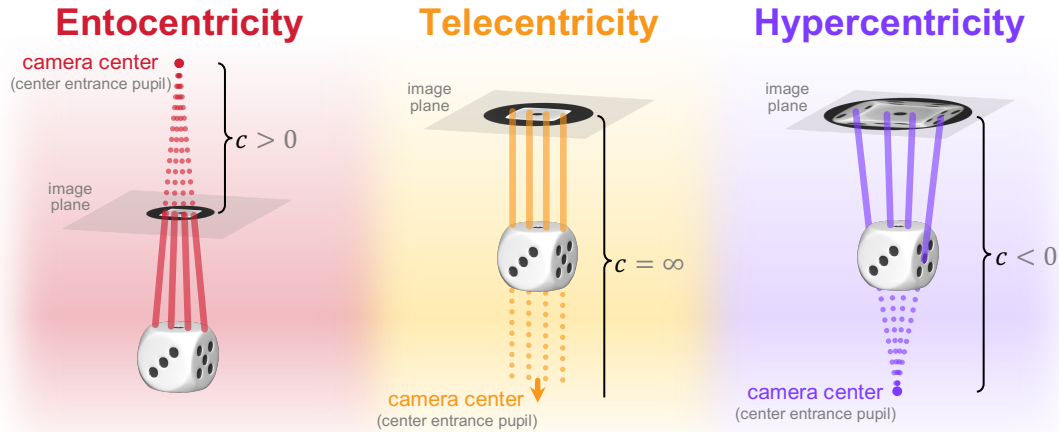


Fig. 2 Object-sided chief ray path and camera model with camera constant c for ento-, tele- and hypercentric lenses. [4]

2. Camera center c :

The camera center is described by

$$\mathbf{c} = \mathbf{e}_n^* - k\mathbf{e}_0^* \quad \text{with} \quad k := 1/c \quad (1)$$

utilizing the *projective geometry* incorporated in the algebra. Due to this formulation, the model has a *smooth transition* (without case differentiation) between *perspective* and *orthographic projection* by varying only one calibration parameter k ("reciprocal camera constant"):

$$k \begin{cases} > 0 \Leftrightarrow \text{entocentric,} \\ = 0 \Leftrightarrow \text{telecentric,} \\ < 0 \Leftrightarrow \text{hypercentric.} \end{cases} \quad (2)$$

3. Camera chief rays:

Given image points \mathbf{p} , the chief rays can be constructed by the algebra's *regressive product*

$$c \vee \mathbf{p} . \quad (3)$$

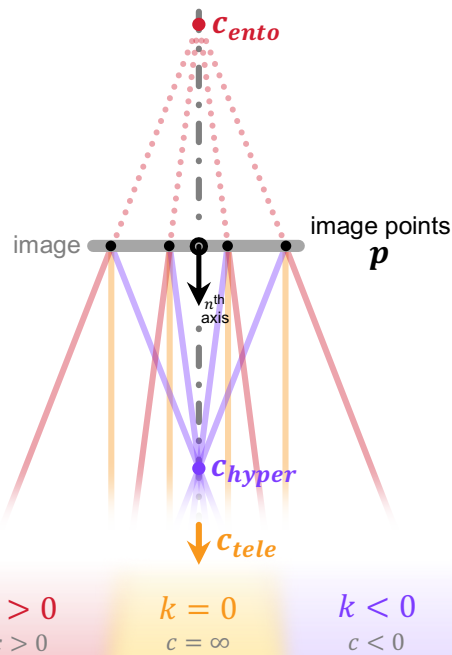


Fig. 3 Anycentric camera model with smooth transition between ento-, tele- and hypercentricity. [4]

4 Conclusion

It was shown how the widely used pinhole camera model can be generalized to encompass ento-, tele- and hypercentric lenses using *PGA*, which can be easily extended by additional common calibration parameters like *distortion*. The model contains a smooth transition between perspective and orthographic projection, making it well suited for the optimization process during camera calibration and resulting in a more complete and more general camera model without greatly increasing its complexity. Furthermore, due to the smooth transition, it provides a highly practical way to directly *calibrate telecentricity errors* of telecentric lenses.

5 References

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