# Modeling light distributions

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In illumination optics, it is often useful to *approximately* describe light distributions in the spatial and angular domains, in order to be able to quickly calculate beam properties analytically. In our paper we discuss distributions of Gaussian and $cos^n$ shape, their advantages and disadvantages and show their application in some examples.

## 1 Introduction

There are various ways to specify a beam that is shining to infinity or at a surface. Customers sometimes prefer a qualitative prescription ("the beam should be nice and bright"), laser people would rely on the Gaussian model. In the automotive world, there are some test points with a specified intensity defined. I all cases it is difficult to provide an immediate estimate for e.g. the total flux of such a beam. Consequently, there's a need for *useful* beam models.

## 2 Angular Beam Models

A first figure of merit to characterize the shape of a beam is the collimation strength which is the ratio between peak intensity and total flux.

$$\kappa = \frac{I_0}{\Phi} \tag{1}$$

which is often indicated on light distribution curves of luminaires.

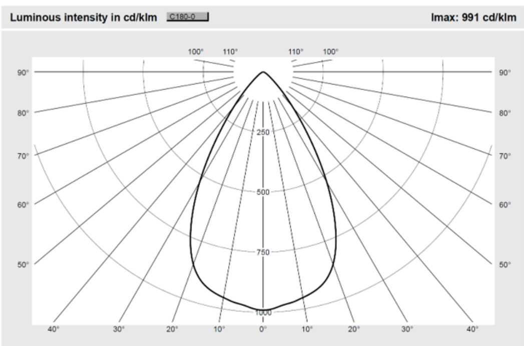


***Fig. 1*** *Collimation strength of a interior light*

The angular shape of the beam may be described by a Gaussian function

$$I(\vartheta) = I_0 \exp\left(-2\,\frac{\sin^2\vartheta}{\sin^2\theta}\right) \tag{2}$$

or a $\cos^n$ distribution:

$$I(\vartheta) = I_0 \cos^n\vartheta \tag{3}$$

In contrast to the former, the latter can be easily integrated

$$\Phi = \int_0^{90^\circ} I(\vartheta)d\Omega = 2\pi I_0 \int_0^{90^\circ} \cos^n(\vartheta)\sin\vartheta\, d\vartheta \tag{4}$$

and yields the relation to the total power of the beam

$$\Phi = 2\pi sr I_0 \left[-\frac{\cos^{n+1}\vartheta}{n+1}\right]_0^{90^\circ} = \frac{2\pi sr}{n+1} I_0 \tag{5}$$

The exponent is related to the collimation strength by

$$n = 2\pi sr\frac{I_0}{\Phi} - 1 = 2\pi sr\kappa - 1 \tag{6}$$

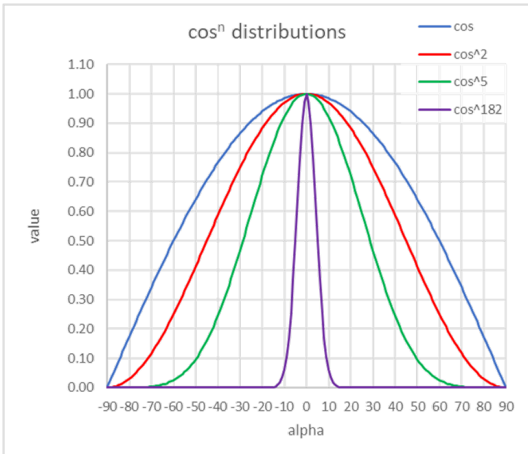


***Fig. 2*** *Various $cos^n$ distributions*

Moreover, we can easily calculate the *full width at half maximum* (FWHM) angle $\theta_F$ (in the world of stage lighting called the *beam angle*)

$$\cos^n\frac{\theta_F}{2} = 0.5 \tag{7}$$

and the exponent via

$$n = \frac{\ln\ 0.5}{\ln\cos\theta_F} \tag{8}$$

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**LED specs example** A certain LED is specified by its FWHM angle $\theta = 45°$ and a peak intensity of $I_0 = 8\ cd$. How much flux is contained in the beam ?

We easily obtain $n = 9.2$ and $\Phi = 5\ lm$.

**Par64 NSP lamp** (a legacy tungsten halogen reflector lamp, NSP means *narrow spot*). In this example, the lamp is specified by a flux of $15800\ lm$ and peak intensity of $I_0 = 255\ kcd$. Calculate the beam angle!

With the above mathematics ,we easily obtain

$$n = 2\pi sr \frac{I_0}{\Phi} = 2\pi sr \frac{255\ kcd}{15800\ lm} = 101 \tag{9}$$

$$\cos^{101} \frac{\theta_F}{2} = 0.5 \quad \rightarrow \quad \theta_F = 13.5° \tag{10}$$

## 3 Spatial Beam Models

Can we use the same distributions to describe the illuminance $E$ on a screen ? This is needed for the specification of surgical lights.

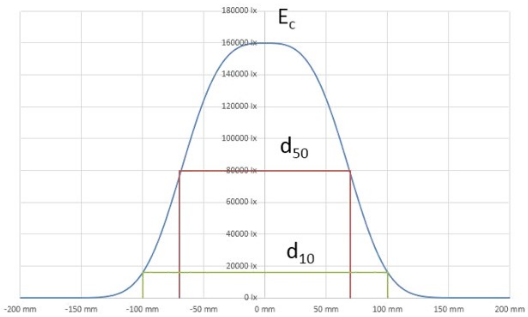


***Fig. 3*** *Illuminance distribution of a surgical light*

We immediately find out that a $cos^n$ shape

$$E(r) = E_0 \cos^n \left(\frac{r}{w}\right) \tag{11}$$

cannot be trivially integrated

$$\Phi(R) = \int_0^R E(r) 2\pi r dr \tag{12}$$

In contrast, the Gaussian distribution

$$E(r) = E_C \exp\left(-2\frac{r^2}{w^2}\right) \tag{13}$$

can be integrated

$$\Phi(R) = \int_0^R E(r) 2\pi r dr = E_C \frac{\pi \cdot w^2}{2} \left[-\exp\left(-2\frac{r^2}{w^2}\right)\right]_{r=0}^R \tag{14}$$

with the total flux

$$\Phi = E_C \frac{\pi \cdot w^2}{2} \tag{15}$$

We may define the spatial collimation strength $\gamma$ as the ratio between center illuminance $E_C$ and total flux

$$\gamma = \frac{E_C}{\Phi} = \frac{2}{\pi \cdot w^2} \tag{16}$$

In a similar way as above we obtain the FWHM diameter of a Gaussian distribution

$$D_F = 2w\sqrt{-2 \ln 0.5} \tag{17}$$

which is used in the lamp specification standard together with the center illuminance.

## 4 Comparison

We close with the statement that Gaussian and $\cos^n$ approximations of a given beam can be very close, especially for narrow beams.

You can find more details and more examples in [1] and more functions in [2].

## References


[1] H. Rehn and J. Muschaweck, *Engineering Illumination Optics. Formulas and Exercises.* (Wiley, 2025).

[2] J. Werner, M. Zhao, M. Hillenbrand, and S. Sinzinger, "RBF-based Optical Surfaces," in *DGaO Proceedings* (2012). URL `https://nbn-resolving.org/urn:nbn:de:0287-2012-P039-`